**Term End External Examination 1ST Semester (Session-Feb 2025)**

**Subject: Mathematics**

***Course No and Title: MMT122M/ Calculus-I***

**Time: 2.15 hours** **Max Marks:100** **Min. Marks:40**

## Section A: Objective Type Questions

**Q1. Choose the appropriate Answer: (8x1.5=12)**

**i.** $\lim_{x\to 0}\left(\frac{\sin ax}{\sin bx}\right)^k$ **equals:**

**A** $\left(\frac{a}{b}\right)^k$ **B** $\frac{a}{b}$

**C** 0 **D** 1

**ii.** **The function** $f(x) = |x|$

**A** Continuous as well as derivable at $x = 0$ **B** Continuous but not derivable at $x = 0$

**C** neither Continuous nor derivable at $x = 0$ **D** None of these

**iii.** **The polar sub-tangent is equal to**

**A** $r\frac{d\theta}{dr}$ **B** $r\frac{dr}{d\theta}$

**C** $r^2\frac{d\theta}{dr}$ **D** $r^2\frac{dr}{d\theta}$

**iv.** **The curvature of the straight line is**

**A** 0 **B** Infinite

**C** 1 **D** None of these

**v.** **Rolle's theorem is applicable for the function** $y = \tan x$ **in the interval**

**A** $0 < x < \pi$ **B** $0 \le x \le \pi$

**C** $-\pi \le x \le \pi/2$ **D** None of these

**vi.** **I** The Maclaurin's series expansion of $e^x$ is

**A** $e^x + e^{2x} + e^{3x} + \ldots$ **B** $1+1+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots.$

**C** $1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+\ldots$ **D** $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots$





**vii.** **What is the relation between evolutes and envelopes**

**A** Evolutes and envelopes are same **B** Evolute is the envelope of normals to a curve

**C** Evolute is the envelope of tangents to a curve **D** Envelope is the evolute of normal to a curve

**viii.** **Envelope of a family of curves can be defined as**

**A** A curve which is a straight line **B** A curve which touches each member of the family of curves

**C** A curve which surrounds the family of curves **D** NONE

## Section-B: Descriptive Type Questions (Short Type)

**Q2: Answer all the Questions (8 x 4 =32)**

**i.** Evaluate $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n+5}$

**ii.** If u= $x^3 + y^3 + 3xyz$, how $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 3u$

**iii.** Find the asymptote parallel to axis of the curve $x^3 + y^2x + xy$

**iv.** Find the angle between the radius vector and the tangent of the curve r= a$\theta$ at any point?

**v.** Verify Role's Theorem for $f(x) = sinx$ in the [0, 2π]

**vi.** Define Bounded function with an example.

**vii.** Explain the term envelope of the family of curves?

**viii.** Find the envelope of the family of curves $y = mx + \sqrt{a^2m^2 + b^2}$ where m is the parameter

## Section – C: Descriptive Type Questions (Medium Type)

**Answer all the questions: (4 x 7=28)**

**Q3.** Find the n-th derivative of $x^3 sinax$

**OR**

State and Prove Euler's theorem.

**Q4.** Find the length of the polar subtangent, polar sub normal of the curve r= $a$sin2Φ, at Φ = $\frac{\pi}{6}$.

**OR**

Find the curvature of $y^2 = 4ax$

**Q5.** Expand $f(x) = \sin x$ in the powers of $(x - \frac{\pi}{2})$.

**OR**

Find the value of 'c' of the function $f(x) = x(x-1)(x-2), [0, \frac{1}{2}]$ by mean value theorem.

**Q6.** Trace the curve whose parametric equations are $x = a\sec\theta$ and y = b tan$\theta$.

**OR**

Find the envelope of the family ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where the two a, bare connected by the relation a + b = c, c being a constant.

**Section – D: Descriptive Type Questions (Long Type)**

**Answer any two of the following: (2 x 14=28)**

**Q7.** State and prove Leibnitz's product theorem?

**Q8.** Find the radius of curvature at any point for the curve $x = a(\theta - \sin\theta), y = (1 - \cos\theta)$.

**Q9.** Expand $f(x) = \log \sec x$ by Maclaurin's theorem.

**Q10.** Find the envelope of the family of ellipses, $x = a\sin(\theta - \alpha)$ and $y = b\cos\theta$, where α is the parameter.